Universal Features in the Growth Dynamics of Complex Organizations

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(Received 3 April 1998; revised manuscript received 9 June 1998)

We analyze the fluctuations in the gross domestic product (GDP) of 152 countries for the period 1950–1992. We find that (i) the distribution of annual growth rates for countries of a given GDP decays with "fatter" tails than for a Gaussian, and (ii) the width of the distribution scales as a power law of GDP with a scaling exponent $\beta \approx 0.15$. Both findings are in surprising agreement with results on firm growth. These results are consistent with the hypothesis that the evolution of organizations with complex structure is governed by similar growth mechanisms. [S0031-9007(98)07339-6]

PACS numbers: 02.50.Ey, 01.75.+m, 05.40.+j, 05.45.+b

In the study of physical systems, the analysis of the scaling properties of the fluctuations has been shown to give important information regarding the underlying processes responsible for the observed macroscopic behavior. In contrast, most studies on the time evolution of economic time series have concentrated on average growth rates [1–18]. Here, we investigate the possibility that the study of fluctuations in economics may also lead to a better understanding of the mechanisms responsible for the observed dynamics [19–23].

We therefore analyze the fluctuations in the growth rate of the gross domestic product (GDP) of 152 countries during the period 1950–1992 [24]. We will show that (i) the distribution of annual growth rates for countries of a given GDP is consistent for a certain range with an exponential decay, and (ii) the width of the distribution scales as a power law of GDP with a scaling exponent $\beta \approx 0.15$. Both findings are in surprising agreement with results reported on the growth of firms [25–27].

It is not obvious that firms and countries show similarities other than that they are complex systems made up of interacting individuals. Hence, our findings raise the intriguing possibility that similar mechanisms are responsible for the observed growth dynamics of, at least, two complex organizations: firms and countries.

We first study the distribution $p(\log G)$, where G is the value of the GDP detrended by the global average growth rate, for all the countries and years in our database. As shown in Fig. 1, $p(\log G)$ is consistent with a Gaussian distribution, implying that $P(G)$ is lognormal. We also find that the distribution $P(G)$ does not depend on the time period studied.

Next, we calculate the distribution of annual growth rate $r_1 \equiv \log[G(t+1)/G(t)]$, where $G(t)$ and $G(t+1)$ are the GDP of a country in the years t and $t+1$. In the limit of small annual changes in G, $r_1(t)$ is the *relative* change in G. For all countries and all years, we find that the probability density of r_1 is consistent, for a certain range of $|r_1|$, with an exponential decay (see Fig. 2a)

$$\rho(r_1) = \frac{1}{\sqrt{2}\,\sigma_o} \exp\left(-\frac{\sqrt{2}\,|r_1 - \bar{r}_1|}{\sigma_o}\right), \tag{1}$$

where σ_o is the standard deviation. We find that the functional form of the distribution is stable over the entire period considered; i.e., we find the same distribution for all time intervals.

We then investigate how the growth rate distribution depends on the initial value of the GDP. Therefore, we divide the countries into groups according to their GDP. We find that the empirical *conditional* probability density of r_1 for countries with approximately the same GDP is also consistent in a given range with the exponential form

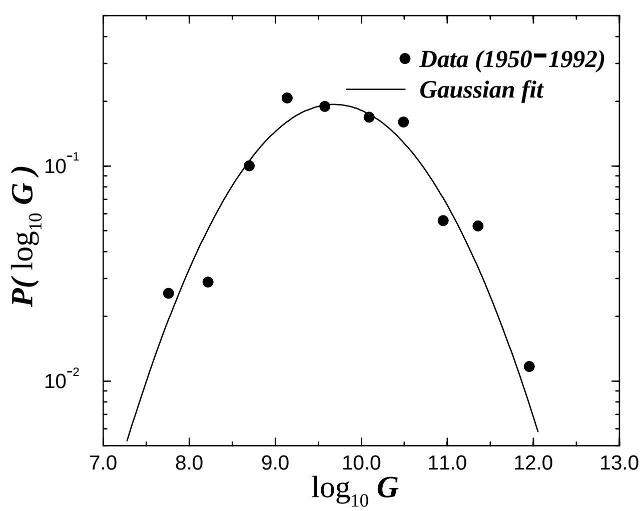

FIG. 1. Histogram for the logarithm of the GDP in units of 1985 international dollars. The data have been detrended by the average growth rate, so values for different years are comparable. The data points are the average over the entire period, 1950–1992, and the continuous line is a Gaussian fit to the data. We also confirmed that the distribution is stationary—i.e., remains the same for different time intervals. The bins were chosen equally spaced on a logarithmic scale with bin size 0.495.

(see Fig. 2b)

$$\rho(r_1|G) = \frac{1}{\sqrt{2}\,\sigma(G)} \exp\left(-\frac{\sqrt{2}\,|r_1 - \bar{r}_1|}{\sigma(G)}\right), \quad (2)$$

where $\sigma(G)$ is the standard deviation for countries with GDP equal to G. Using a saddle point approximation, we may integrate the distribution (2) over $P(G)$ using a lognormal distribution and recover (1).

Figure 3a shows that $\sigma(G)$ scales as a power law

$$\log \sigma(G) \sim -\beta \log G, \quad (3)$$

with $\beta \approx 0.15$. We confirm our results by a maximum-likelihood analysis [28]. In particular, we find that the log-likelihood of $\rho(r_1|G)$ being described by an exponential distribution—as opposed to a Gaussian distribution—is of the order of e^{600} to 1. Similarly, we test the log-likelihood of σ obeying (3). We find that Eq. (3) is e^{130} more likely than $\sigma(G) =$ const, and that adding an additional nonlinear term to (3) does not increase the log-likelihood.

The results of Figs. 1–3 are in *quantitative* agreement with findings for the growth of firms [25–27]. Figure 4a shows that the same functional form describes the probability distribution of annual growth rates for both the GDP of countries and the sales of firms [29]. Moreover, as shown in Fig. 4b, the width of the distribution of annual growth rates also decays with size with the same exponent for firms and countries.

We test the hypothesis that the growth rates of firms and countries are described by the same probability distribution. We use the Kolmogorov-Smirnov (KS) test [28], which defines a measure of the difference D between

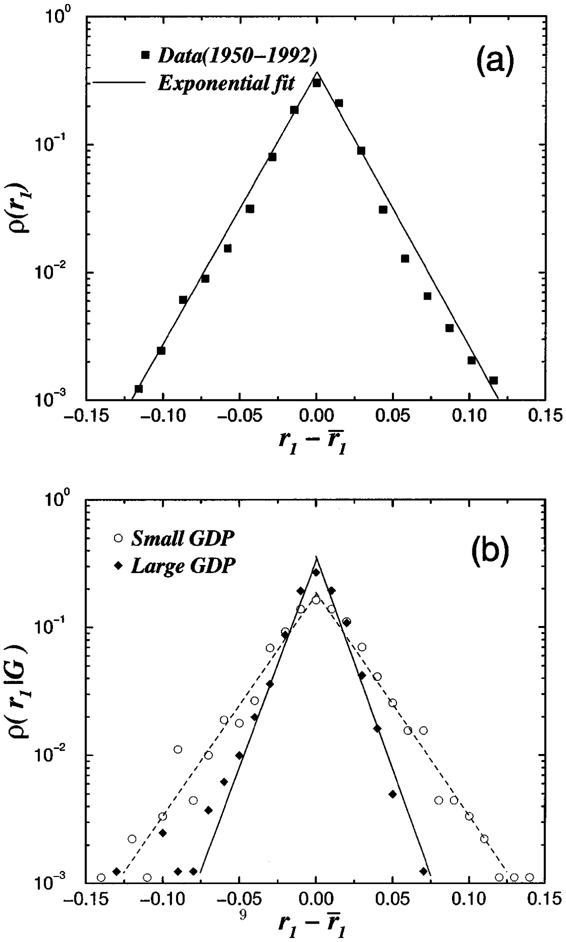

FIG. 2. (a) Probability density function of annual growth rate r_1. Shown are the average annual growth rates for the entire period 1950–1992 together with an exponential fit, as indicated in Eq. (1). (b) Probability density function of annual growth rate for two subgroups with different ranges of G, where G denotes the GDP detrended by the average yearly growth rate. The entire database was divided into three groups: $6.9 \times 10^7 \leq G < 2.4 \times 10^9$, $2.4 \times 10^9 \leq G < 2.2 \times 10^{10}$, and $2.2 \times 10^{10} \leq G < 7.6 \times 10^{11}$, and the figure shows the distributions for the groups with the smallest and largest GDP. We consider only three subgroups in order to have enough events in each bin for the determination of the distribution.

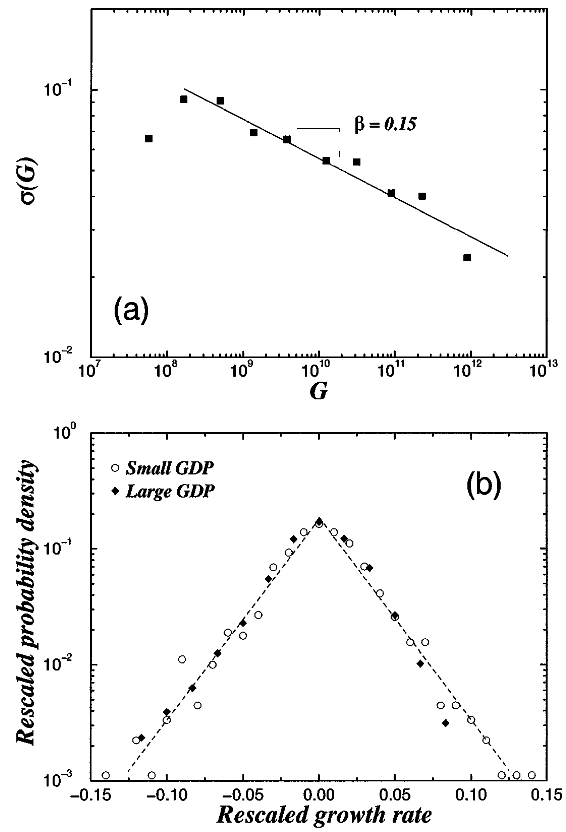

FIG. 3. (a) Plot of the standard deviation $\sigma(G)$ of the distribution of annual growth rates as a function of G, together with a power law fit (obtained by a least squares linear fit to the logarithm of σ vs the logarithm of G). The slope of the line gives the exponent β, with $\beta = 0.15$. The bins are the same as in Fig. 1. Note that statistics for different bins is different as can be seen in Fig. 1. Also note that we use here more bins than in Fig. 2b, because we need fewer points per bin for the determination of the standard deviation than for the determination of the distribution. (b) Rescaled probability density function, $\sigma(G)\rho(r_1|G)$, of the rescaled annual growth rate, $(r_1 - \bar{r}_1)/\sigma(G)$. Note that all data collapse onto a single curve.

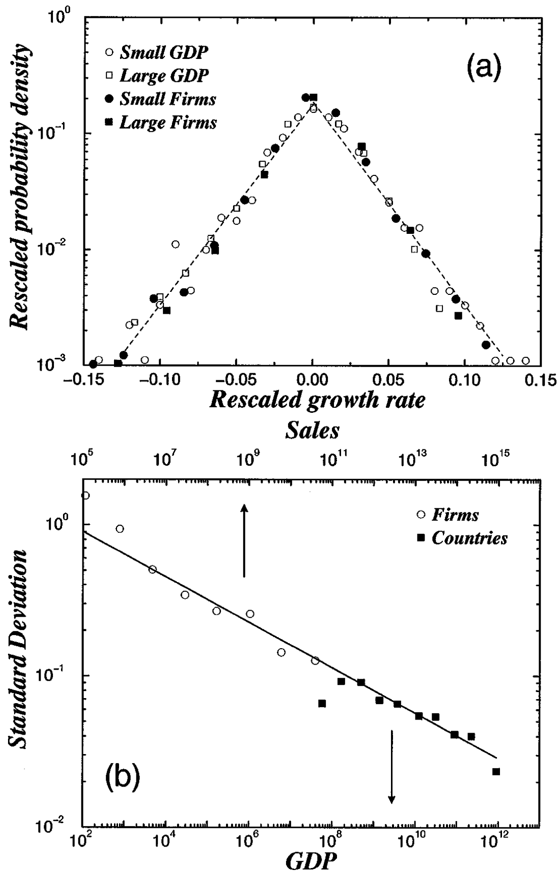

FIG. 4. Test of the similarity of the results for the growth of countries and firms. (a) Conditional probability density of annual growth rates for countries and firms. We rescale the distributions as in Fig. 3b. All data collapse onto a single curve showing that indeed the distributions have the same functional form. (b) Standard deviation of the distribution of annual growth rates. Note that σ decays with size with the *same* exponent for both countries and firms. The size is measured in sales for the companies (top axis) and in GDP for the countries (bottom axis). The firm data are taken from the COMPUSTAT database for publicly traded manufacturing firms from 1974–1993 (see [26] for details).

the empirical distribution functions of the data sets for sales and GDP. For a given measured value of D, one estimates the probability p_{KS} that the difference is at least as large as D under the assumption that the data sets are drawn from the same probability distribution.

The KS test requires certain conditions that are not obeyed by the data. In particular, the growth rates are subject to different measurement errors that are larger for the GDP. Moreover, the growth rates are correlated over time and among countries and firms, effectively decreasing the number of independent measurements. To reduce the correlations, we select random samples with 10% of the number of points we have for countries and firms. Before applying the KS test, we normalize all growth rates; for countries, we use the transformation $r \equiv (r_1 - \bar{r}_1)G^{\beta}$, and, for firms, we use $r \equiv (r_1 - \bar{r}_1)S^{\beta}$. This normalization allows us to consider in the test the growth rates of firms and countries with different sizes.

We find $p_{\mathrm{KS}} = 0.1$, which means that the Kolmogorov-Smirnov test *cannot reject* the hypothesis that the two distributions are the same at the usual significance level of 5%. Hence, for all practical purposes, the data are consistent with the assumption that the two distributions for sales and GDP are identical.

If the *same* empirical laws hold for the growth dynamics of both countries and firms, then a *common* mechanism might describe both processes. To explore this possibility, we consider two limiting models.

(i) Assume that an economic organization, such as a country or a firm, is made up of many units, which are of identical size and grow independently of one another. Then, the growth fluctuations as a function of size decay as a power law with an exponent $\beta = 0.5$. This result is due to the fact that the number of units forming a given organization is proportional to its size, and because the variance of the sum of n independent quantities grows like $\sqrt{n}$ [26].

(ii) Assume that there are very strong correlations between the units, which is the opposite limiting case. Then, it follows that the growth dynamics are indistinguishable from the dynamics of structureless organizations. As a result, we obtain an exponent $\beta = 0$; i.e., there is no size dependence of σ.

The fact that the exponent β for the empirical data is between the two limiting cases shows that the models (i) and (ii) are both based on false assumptions. Our results are consistent with a recently proposed model [30] for the growth of organizations. The dynamics of the model give rise to subunits whose characteristic size increases with the size of the organization leading to an exponent β smaller than $1/2$.

Our empirical results suggest an important consequence for economic growth: Although large economies tend to diversify into a wider range of economic activities leading to smaller relative fluctuations, the degree of diversification observed is much smaller than what would be expected if diversification would increase linearly with the size of the economy—which would correspond to $\beta = 0.5$. This effect is quantitatively the same for firms and countries, which raises the intriguing possibility that a *common* mechanism might characterize the growth dynamics of economic organizations with complex internal structure. The existence of "universal" mechanisms, which can give rise to general laws that are independent of the particular details of the system, could provide a firmer grounding for the application of physics methods to questions in economics [19–23,31].

We thank E. Alexander, S. Alexander, S. V. Buldyrev, Y. Liu, P. Gopikrishnan, V. Horváth, R. N. Mantegna, C.-K. Peng, M. A. Salinger, and especially S. Havlin and J. D. Sachs for helpful discussions. Y. L. thanks the Korea Research Foundation for support, L. A. N. A. thanks

JNICT (Portugal), and M. M. thanks the DFG. The CPS is supported by grants from NSF and NIH.

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